Journal of Mathematical Sciences 2000 vol.99 N3, pages 1348-1366

On limit distributions emerging in the generalized Birnbaum-Saunders model

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Abstract

We establish that the Birnbaum-Saunders distribution is the equilibrium mixture of the inverse Gaussian distribution and the convolution of this distribution with the chi-square distribution with a single degree of freedom. We give a physical interpretation of this phenomenon in terms of probabilistic models of fatigue life and introduce a general family of so-called crack distributions, which contains, in particular, the normal distribution, the inverse Gaussian distribution, and the Birnbaum-Saunders distributions, as well as others used in applications of the theory of reliability. We pose the problem of isolating these particular distributions lying on the boundary of the parametric space of a crack distribution; to solve this problem, we analyze the asymptotic behavior of the likelihood function of the maximal invariant for a random sample of a crack distribution as the sample size n grows and for large values of the form parameter λ, when the crack distribution is approximated by the normal distribution. In either case, the likelihood function asymptotically depends on the sample data only through the U-statistics ∑i=1 n Xi ∑i=1 n Xi -1. This result allows us to construct asymptotically uniformly most powerful invariant tests for n → ∞ or λ → ∞ the latter "parametric" scheme of asymptotic analysis of the likelihood is similar to LeCam's theory of statistical experiments, where large values of n are formally replaced by large values of λ, whereas the sample size n remains fixed. © 2000 Kluwer Academic/Plenum Publishers.